Faculty Working Papers

MODELING THE MARKETING MIX

FOR AN INDUSTRIAL PRODUCT

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Introduction

Although marketing mix problems for consumer goods producers have been extensively modeled in the marketing literature, no corresponding modeling effort has yet taken place for the industrial products case. Since the customers in the consumer and the industrial products markets can be expected to differ in several respects, it is clear that models developed for the former market will not generally be of much use in the latter case. With industrial marketing coming into focus in recent years, especially as concerns industrial buyer behavior--for an up-to-date treatment, see (10)--the groundwork has been laid for a well-founded modeling of the marketing mix problem facing the industrial products seller. This paper develops one version of such a marketing mix model.

In what follows the extant theory and empirical findings of industrial buyer behavior will first be surveyed. On this basis the likely customer responses to the available instruments will then be developed. This development will be made more exact by translating the a priori verbal relationships into mathematical representations incorporating the essence of the stated relations. The model incorporates both stochastic and deterministic elements, in that the parameters of the utilized probability distributions are modeled deterministically. After the general form of the optimal decision rules are developed for the profit

maximizing case, an example is presented showing a concrete application of the model.

Some Major Features of Industrial Buying Behavior

One basic finding in industrial buying research has been that the purchase decision maker is rarely a single person. Rather, several persons, including a purchasing agent and an engineer and/or user of the product, will make up a more or less loose group of decision makers [3, 10]. On the other hand, it has been established that this group decision making occurs sequentially, rather than simultaneously. Thus, the final decision to purchase from a deliverer rests usually with the purchasing agents. Which firms/products to consider, however, is to a large extent determined by the engineers' quality tests and the users' experience with the product. A bad experience qualitywise or servicewise, or simply in terms of delivery days, will generally be communicated in negative terms to the purchasing agent, sometimes making a repeat purchase impossible [3].

Accordingly, once a firm's product has been tried, the experiences with it will tend to dominate in the determination of a possible return to the same firm. The purchasing agent will still have some leeway, however. He is basically in charge of identifying new suppliers [3, 11]. Furthermore, when a new product is needed, any of the competing sellers whose offering fulfills the basic engineering/user requirements will be eligible. Finally, in many cases the performance of the present product will be merely "satisfactory" and other firms' products might be tried. The Scientific American study found that whereas the initial specifications desired from the product were within the purchasing

agent's purview only 7% of the time, he was solely in charge of deciding upon final supplier in 67% of the cases. The basic consideration in the latter decision between acce table sellers seems to be price and shipping costs [3].

In identifying potential suppliers, the purchasing agent relies on past experience, and also on the suppliers' salesmen and advertising. Although the salesmen clearly would like to have access to all partners in the purchasing decision group, Levitt found that the purchasing agent functioned as a "filtering" device, prohibiting most salesmen from ever reaching the engineers and ultimate product users [6]. Similarly, advertising information, in catalogues, direct mail, and trade journals, tends to reach only the purchasing agent, if any one at all [6, 11].

Since the agent often is making his decisions under various constraints, including time limitations, only a relatively small number of firms can be contacted for price and shipping costs quotations [11]. This provides for one possible difference between the personal selling approach and incomplete personal information, since in the former case such quotations will be immediately available. On the other hand the different kinds of advertising will serve as a cheap means of making the purchasing agent aware of the supplier. Although no empirical study has yet been carried out for the industrial buying case, we know from the buyer behavior theory that an increase in the evoked set of suppliers-- will tend to lead to an increase in search activity [10]. Thus, advertising in any of the forms described might be justified.

The overall quality judgment of a tried product will be a function of many factors. Apart from actual performance in production, service

requirements and delays, in addition to delivery times and design questions, and even the calling salesman's know-how, will be of importance [6]. Furthermore, the overall rating will rarely depend upon one person's judgment, but both quality control and production engineers will figure prominently in any composite judgment [10]. In addition, there is some evidence that word-of-mouth communication can be high between different companies, making it possible for the engineers to form quality opinions without actually trying out the product [4].

Modeling Implications

This discussion of industrial buying behavior should in no way be regarded as complete. Rather, we have deliberately focused upon those parts of the theory and empirical findings which most directly generate an understanding of the possible responses to our supplying firm's decision variables of price, quality, personal selling, and advertising. Let us then extract the implications of that discussion for the modeling of the marketing mix which is of course our main concern in this paper.

First, it seems reasonable to assume that over time the most important marketing mix variable becomes the quality of the product. At an introductory stage, the advertising and personal selling efforts will perhaps pay off by increasing the awareness of the product and thereby increasing the trial rate. As time goes on, however, these trials and the word-of-mouth communication generated will gradually make quality experiences of highest importance.

Second, once quality ratings have been firmly established, a change in product quality cannot be made easily or, at least, not communicated to the buyers very easily. Even if a change is made, and advertising

and personal selling employed to inform the market of it, this communication will only slowly filter through to the engineers and users responsible for quality judgments. Accordingly, a change in quality can for most practical purposes be seen as an introduction of a new product.

Third, after quality ratings have been used to establish a list of "eligible" suppliers, the final choice of product will be made on the basis of price (including shipment costs). Again, in the initial stages, the list might be increased to include as yet untried firms' products, for example, through the use of advertising or personal selling. To the extent word-of-mouth communication is functioning, however, some of these untried products might be deleted on the basis of reputed low quality.

Fourth, personal selling differs from advertising in the amount of information yielded and the role in product servicing. On the other hand, it is questionable whether advertising has any effect which could not be achieved through the use of salesmen (although it might be a cheaper way in some cases). Because of the "spillover" effects of personal selling efforts on quality ratings, it is understandable that much industrial selling is done through salesmen. Conversely, advertising is often seen as a supportive activity to the salesman's efforts [1, Ch's 14, 15]. Accordingly, there are reasons for defining the personal selling variable as including a certain proportion of advertising, and treat personal selling and advertising as one decision variable.

Fifth, the overall judgment of the product quality represents a summary measure of the product's ratings on several dimensions. Although each of these dimensions consitute a decision variable in its own right

(service, design, delivery decisions, etc.) for the purposes of the present model they can all be subsumed under the overall quality decision variable. The main reason is that the overall quality judgment is what matters to the buyers, and thus is the interesting variable here. It should be kept in mind, however, that there exists a "quality mix" problem of positioning the product on the quality dimensions which is disregarded in this treatment.

Model Preliminaries

In what follows, these considerations will be employed in the development of a model of the effects of the three decision variables quality, price, and personal selling upon the supplying firm's sales and profit figures. Before the model can be developed, however, some specific assumptions about the industry have to be made.

These assumptions are that:

(A) The number of customers in the market remains fixed at N throughout the period of analysis (no growth in the market).

(B) For the current period the supplying competitors do not know each other's prices, but they know the prices quoted in the past.

(C) The demand is such that each demand period is clearly discernible (e.g., seasonal demand).

(D) No quantity discounts are offered.

The most restrictive assumption is clearly (C). Should (C) be untenable, it could be replaced by the assumption that only a fraction of the total number of customers buys each time period. If this fraction stays relatively stable over time, the effect would simply be to reduce the total market available in any one period. Similarly, there

are fairly easy ways of adapting the other assumptions if the conditions in the particular application necessitate it.

First Period Modeling: The Basic Model

Let N be the total number of customers in the market. The probability that "our" supplying firm is on the acceptable supplier list of n of these customers, $n \leq N$, is denoted by $F(n)$. In the first period, this probability depends mainly upon our personal selling effort (S_o) since word-of-mouth communication is non-existent. Let the probability that any one of these n customers will buy from us equal σ_{p_o} which depends upon our price (p_o). Then the probability of r customers buying out of n listings $r \leq n$, can be seen as a binomial distribution, and

$$(1)\quad \text{Prob } (r \text{ and } n) = \text{Prob } (r/n) \times F(n)$$

$$= \binom{n}{r} \sigma_{p_o}^{\ r} (1-\sigma_{p_o})^{n-r} \times F(n).$$

Let the total number of competitors equal m. The number of customers to list our firm (n) can be represented as a random variable with a Poisson distribution. The mean of (n), call it λ, will in the absence of personal selling and quality differences between competitors be equal to N/m. Then

$$(2)\quad F(n) = \frac{e^{-\lambda}\lambda^{n}}{n!}\ .$$

From a knowledge of σ_{p_o} and λ, the expected sales for the firm can be calculated.[1] Accordingly, our subsequent modeling will focus on the explanation of these two parameters.

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[1]This basic model is essentially equivalent to the one presented in [9, pp. 34-35.

<u>First Period Modeling. The Role of Personal Selling</u>

When personal selling is carried out by any of the firms in the industry, the main effect is to increase the awareness and the search activity by the customers. Accordingly, the salesmen expenditures will raise the level of k, a constant which measures the average number of suppliers listed by each customer ($k \geq 1$). Constraints upon the purchasing agent will make him somewhat reluctant to add new firms to the list, however. What matters then is not the actual dollars spent on personal selling but rather the share relative to competing suppliers. Hence, with personal selling by our firm denoted by S_o and personal selling by all the competitors together by S_c, we have

$$(3) \quad \lambda = NgKk \frac{S_o}{S_o + S_c} \quad .$$

The relationship between k and the total industry spending has the characteristics of an S-shaped curve. It reaches a saturation level at $k_s = m_1 + \frac{m - m_1}{m}$, where $m_1 \leq m$ denote the number of competitors that do personal selling. This saturation level is derived from the fact that at the limit no firm can get more than N listings. The reason for an initial "threshold" is the belief that very low levels of salesmen's efforts will lead to no marked increase in k--the salesman may not even get to meet the agent, for example. The threshold should occur relatively early, however, resulting in the skewed sigmoid of Figure 1.

Figure 1

One possible expression for the function is

$$(4) \quad \frac{k - 1}{k_s - k} = \alpha_0 \, (S_o + S_c)^{\alpha_1},$$

Where α_0, the scale factor, and α_1, the slope determinant, can be determined empirically from data on personal selling effort and length of customers' lists.[2]

The function determining λ has the following desirable properties, easily checked by substituting the requisite values:[3]

1. When $S_o = S_c = 0$, $\lambda = 0$.
2. When $S_o > 0$, $S_c = 0$, $\lambda = N$ (since $k=1$).
3. When $S_o = 0$, $S_c > 0$, $\lambda = 0$.
4. Our firm attracts a portion of the increased number of listings which is proportional to our personal selling "share."

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[2] For a derivation of the characteristics of this function, see [5]. The observations on the number of competitors listed by the customers could be generated through our salesmen.

[3] These properties relate to the first period only. Over time, selling effort will have a diminishing influence as will be seen later. Nevertheless, if no selling effort is made initially by the firm, the model does predict zero listings.

First Period Modeling: The Role of Price

The probability that our firm will make a sale to a customer (σ_{p_o}) is seen as identical to the probability that our firm charges the lowest price among the firms listed by this particular customer. As we know our competitors' past prices, we can develop a frequency distribution of price for each of them. Assuming the past prices to be representative of what prices might be charged in the coming period, we can then compute the probability that any given price of ours will be lower than some competitor's.

Thus, for example, if the frequency distribution of the price of competitor i, say, looks as in Figure 2, we can compute the probability of our price p_o being lower than his as

$$(5)\quad \text{Prob}\ (p_o < p_i) = \int_{p_o}^{\infty} p_i \, dp_i \ .$$

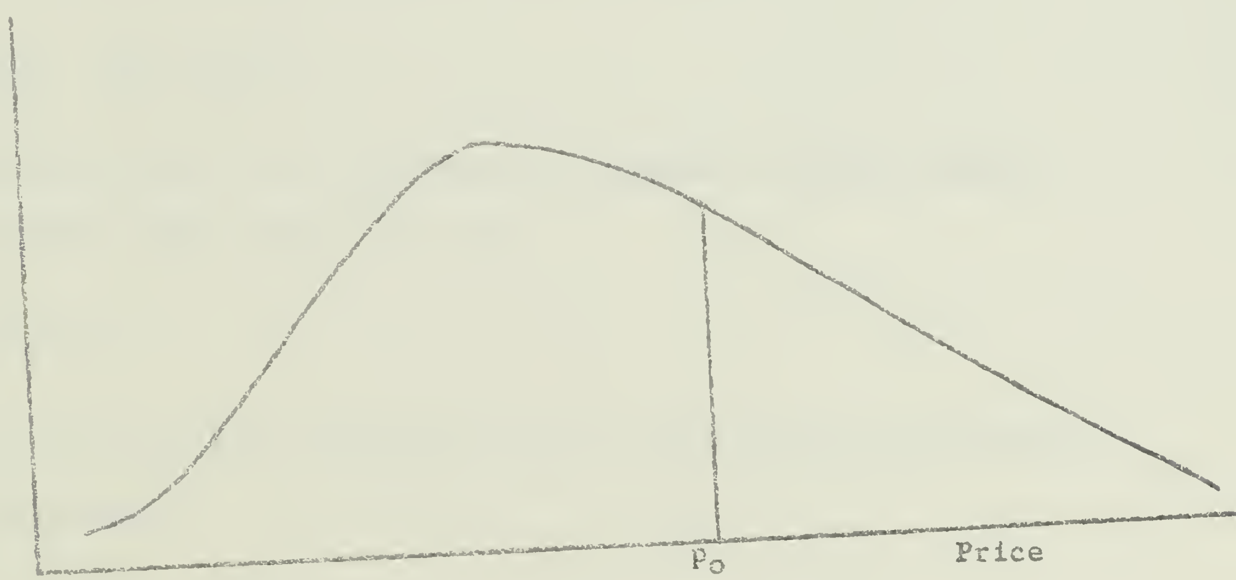

Figure 2

If a particular customer lists k' firms besides ours, the probability that he will buy from us is equal to the probability that our price is the lowest among those $k'+1$ firms. Accordingly, if we renumber these k' competitors 1 through k', the probability of selling to this customer is

$$(6) \quad \text{Prob (1)} = \text{Prob } (p_o < p_i,\ i=1, \ldots, k')$$

$$= \int_{p_1}^{\infty} \int_{p_2}^{\infty} \cdots \int_{p_{k'}}^{\infty} p_1 p_2 \cdots p_{k'} dp_1 dp_2 \cdots dp_{k'} = \sigma_{p_o} .$$

Alternatively, where the competitors listed are hard to identify, an approximate procedure might do. We can compute the probability distribution of average price on the basis of historical data, and then apply this to the average number of competitors listed. This number is equal to k, and depends upon personal selling, as we saw in the last section. Letting $\tilde{\tilde{p}}$ stand for the mean price, we have

$$(7) \quad \sigma_{p_o} = \left(\int_{p_o}^{\infty} \tilde{\tilde{p}}\, d\tilde{\tilde{p}}\right)^k .$$

If the past price data on our competitors is scanty, a further approximation sometimes useful might be to let

$$(8) \quad \tilde{\tilde{p}} = \beta p_o ,$$

where p_o is our own price distribution. The proportionality constant β could be assessed as

β = mean price last period/our price last period.

Second Period Modeling: The Role of Quality

Assume that r number of customers ($r \leq n$) bought from our firm last time period. The portion of these ustomers that will list us again is determined by the quality of our product relative to that of our competitors. As it will be advantageous to have an index of product quality ranging between 0 and 1 we will first derive such a measure. Then the precise effect of quality upon customers will be modeled.

To develop a measure of our quality as perceived by the customers, assume first that the dollars our firm spends on all the quality dimensions amount to q_o. Some of the quality ratings are not directly due to expenditures as such (e.g., delivery timing), but can easily be transformed into such dollar equivalences (this has to be done in many cases for accounting purposes anyway). The mean of our competitors' spending on the corresponding dimensions is denoted by q_c. The overall rating of our quality will most probably be a function of the ratio between q_o and q_c. The relationship will not usually be linear, however, since at the upper limit of q_o/q_c some customers might remain unpersuaded. Thus, the relationship would take the form exhibited in Figure 3.

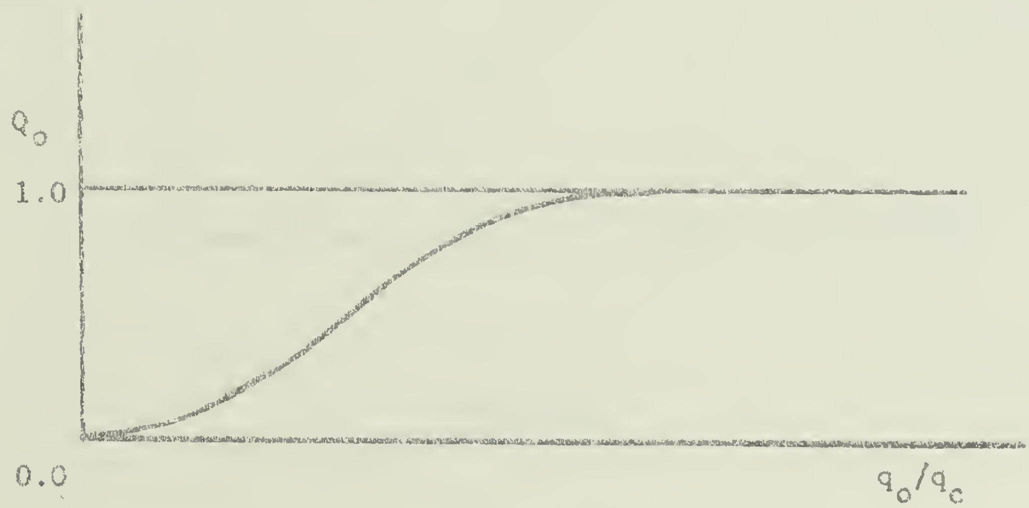

Figure 3

[illegible]

[illegible]

[illegible]

One function that will correctly represent the curve in Figure 3 is the following:

(9) $Q_o = 1.0 - M_e^{-q_o/q_c}$, M a positive constant, perhaps equal to 1.0.

As can be easily verified, this function goes to zero as our quality becomes very low relative to average quality, and it approaches one as our quality becomes very much better than the average. In other words,

$$Q_o \longrightarrow 0, \text{ as } q_o \longrightarrow 0 \text{ and } Q_o \longrightarrow 1, \text{ as } q_o/q_c \longrightarrow \infty.$$

Let the number of purchasers from last period who still list our firm equal r_1 ($r_1 \leq r$). The desired relationship between Q_o and r_1 incorporates the idea of continuously diminishing returns. That is, the biggest loss of customers due to bad quality occurs when the quality is really low. As quality improves, the marginal gain of additional customers becomes gradually smaller. This type of functional relationship is portrayed in Figure 4.

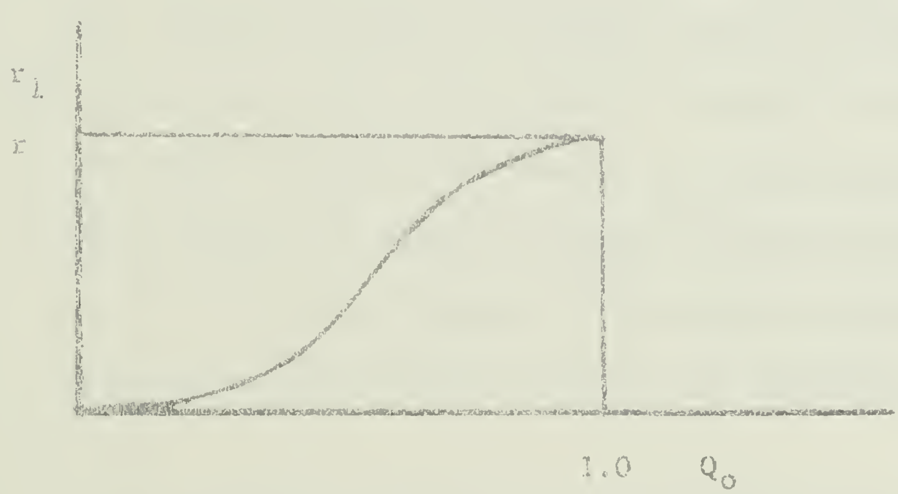

Figure 4

We can represent this curve by

$$(10)\quad \frac{dr_1}{dQ_o} = \frac{K}{Q_o},$$

with $K > 0$ an unknown constant. Then by integrating we get

$$(11)\quad r_1 = K \ln Q_o + C,$$

with C the constant of integration. When $Q_o = 1$, $r_1 = r$, so that $C = r$. The other limiting condition, that the slope of the function becomes very large as Q_o approaches zero, holds regardless of the value of K (as long as $K > 0$). The actual value of K has to be estimated from data on quality ratings and number of returns.[4]

Second Period Modeling. The Total Number of Calls

Quality has also another function, which we will set forth when computing the total number of listings the firm can expect in the second period. Because word-of-mouth communication occurs, product experiences of one customer will at times be communicated to other customers. Thus, the total number of listings that our firm can expect in the second period comes from three different groups of customers. One is the group (with r_1 members) who bought last period and who retains us. Another consists of those customers who have not yet tried the product but who will list us because of the reputed quality. A third group comprises those customers who have been attracted by our

[4]Again, for the quality ratings, our own salesman should be able to generate the necessary observations. As will be seen in the numerical example given later, a few of these "free" parameters, such as M and K, will actually be determined (when other parameters are specified) by the dynamic properties of the model.

firm's personal selling effort. Again, as in the first period, once on the list the probability of purchase is determined by the price.

In modeling the second period λ, the parameter of the Poisson distribution of n, the number of calls, we have already dealt with the r customers that bought last time--r_1 of these will have had positive product experiences. The remaining N-r customers are divided into two groups: those that are affected by the product quality, and those affected by personal selling. The latter group comprises basically those customers who have not received any word-of-mouth communication about our firm's product.

Since a direct measure of the proportions falling into either group would be difficult to get, the following approximation will be useful in most cases. We argue that the amount of word-of-mouth information generated with respect to a product is largely proportional to the number of purchasers the product has. If so, the proportion of the N-r customers who would be influenced by quality would be equal to r/N. Correspondingly, the proportion affected by personal selling would equal (N-r)/N. These proportions will be denoted b_1 and b_2, respectively ($b_1 + b_2 = 1$).

As in the first period, the number of customers from these two groups that actually call upon the firm is determined by its differential advantage relative to its competitors. The specification follows the first period rather closely:

$$(12)\quad \lambda_2 = r_1 + b_1(N-r)k(1-Me^{-q_o/q_c}) + b_2(N-r)k\left(\frac{S_o}{S_o+S_c}\right),$$

the subscript on λ indicating the time period.

In this formulation it is assumed that those customers mainly influenced by quality in their choice of what firms to list maintain the same level of calls as those influenced by personal selling. The number of customers calling consists of three parts, two of which depend on how our quality and personal selling effort rate relative to competition. In addition we here get the "loyal" r_1 customers still listing us.

t'th Period Modeling: How Many Will Return?

When the time periods under consideration are increased above two, the basic question is how many customers will keep listing us. From our preceding modeling we know that once they call, price will determine sales. We also know that personal selling and quality together determine the number of calls. But will long-term loyalties be allowed to be formed, and will there be customers who never list us again?

The answer to the loyalty question lies in the quality effect. In the second period we saw that a portion of those who bought the first time around did return (r_1) because of relative quality. If we allow the same mechanism to work in the third and subsequent periods, there will clearly be a possibility of a loyalty development. True, price will be the ultimate determinant of purchase, but whether or not a customer will list us will be dependent upon the quality. Thus, in any time period, say the t'th, the same high-quality firm will be listed continuously, perhaps in addition to some other firms (if k is greater than 1). These repeated listings constitute the basic loyalty phenomenon in the model.

Turning to the other extreme--whether some customers will never repeat a listing--there are two factors to consider. One, as long as there is at least one satisfactory supplier, a bad experience with one firm's product will probably lead to a maintained elimination of the listing of that firm. On the other hand, since the salesman will have some spillover effects on quality ratings, there would exist some small chance of returning the firm to the list through the salesman's efforts.

To account for both of these factors, the value of b_2--which measures the proportion of customers influenced by personal selling efforts--will be subject to the following exponential decline:

$$(13) \quad b_{2,t+1} = b_{2,t} \times (N-r)/N$$

where r_t denotes the number of customers buying from our firm in period t[5]. Since b_1--the proportion of customers who are influenced by quality considerations--is equal to $1-b_2$, there will take place an increase in those quality conscious which is equal to the decrease in b_2. This represents the fact that bad experiences will lead customers not to list us again, and this particular formulation incorporates as a special case the earlier second period values of b_1 and b_2 (in the first period $b_1=0$, and $b_2=1$).

On the other hand, the exponential decline makes b_2 only reach zero asymptoetically, thus preserving some influence of personal selling over time. This represents the spillover of selling effort upon quality. Again, an actual change in quality will have the effect of

[5] This subscript will be suppressed in what follows when no ambiguity can arise.

moving the firm back to period one, since a change has to be properly communicated to the customers before quality can really be effective.

The Cost Structure

Our firm's total production costs can be represented as

$$(14)\quad PC = a_1^1 + a_2^1 r\ ,$$

where a_1^1 represents fixed costs, and a_2^1 unit variable costs. These variable costs are assumed constant throughout the range of operations. Such an assumption is sometimes unwarranted in real world applications, but the analytical convenience it offers here is considerable.

There are also distribution costs which vary depending upon the customer's geographical location. Since the customer is charged for these costs (shipment costs), different customers will in general be faced with different prices from the same firm. Let the price to the i'th customer be equal to p_{oi}, which incorporates the distribution costs to i. Then the probability that we will sell to i is equal to the probability that our price is lowest among those competitors listed by i. For any one of these competitors, say, the j'th,

$$(15)\quad \text{Prob } (p_{oi} < p_{ji}) = \int_{p_{oi}}^{\infty} p_j dp_j$$

$$= \sigma_{p_{ij}}$$

is the probability that we will "beat" j. Assuming that i lists k_i competitors, the probability of us selling to i can be approximated by

$$(16)\quad \text{Prob } (p_{oi} < \bar{p}_{k_i} i) = \left[\int_{p_{oi}}^{\infty} \bar{p}_{k_i}\, dp_{k_i}\right]^{k_i}$$

$$= \sigma_{p_i}\ ,$$

where the approximation derives from the fact that the average competitive distribution, $\tilde{p}_{k_i}$, is used. Similarly, we can derive a distribution for all the n customers listing us and the different σ_{p_i}'s, i = 1,2,...,n, could be substituted into the expression giving the expected purchase rate (here the correct formula is clearly a Bernoulli trial with shifting probabilities of success, not a binomial distribution).

As a further approximation, it might be useful to use the average price distribution, $\tilde{\tilde{p}}$, of all the m competitors, and then for each p_{oi}, i = 1,2,...,n, compute the appropriate probability of winning that customer, using an average calling frequency k. This approach would still amount to a series of Bernoulli trials, with varying probabilities. To derive one average probability of success, $\sigma_{\tilde{p}_o}$, which is the form of the earliest formulation, one could then finally compute

$$(17) \quad \sigma_{\tilde{p}_o} = \frac{1}{n} \sum_{i=1}^{n} \sigma_{p_{oi}} ,$$

using this approximation in the requisite formula [1].

<u>Expected Profit--1st Period Case</u>

In this section an expression for expected profit based on the preceding model and cost structure of the firm is presented. From this expression values of optimal personal selling expenditures and the optimal price to charge for given quality expenditures are derived analytically. From eq(3), the value of λ, for the 1st period is given by

$$\lambda = N \times K \times \left[\frac{S_o}{S_o+S_c}\right]$$

From eq(1) and eq(2)

$$\text{Prob (n listings and r sales)} = \frac{e^{-\lambda}\lambda^{n}}{n!} \binom{n}{r} (\delta_{p_o})^{r} (1-\delta_{p_o})^{n-r}$$

$$= \text{Prob (n and r)}$$

Expected number of customers that will buy - E(r)

$$E(r) = \sum_{n=o}^{n} \sum_{r=o}^{n} \text{Prob (n and r)} \times r$$

$$(18) \quad E(r) = \sum_{n=o}^{n} \sum_{r=o}^{n} \frac{e^{-\lambda} \cdot \lambda^{n}}{n!} \binom{n}{r} \left(\delta_{p_o}\right)^{r} \left(1-\delta_{p_o}\right)^{n-r} \times r$$

If each customer is charged price p_o, the Expected Sales would be,

$$(19) \quad E\ (\text{Sales}) = E(r) \times p_o$$

Total Personal Selling Costs of the Firm in 1st Period = S_o

Total Quality Expenditures = q_o

Expected Costs of Production = $a_1 + a\,E(r)$

where a_1 and a are constants, unique to each firm.

$$(20) \quad \text{Expected Costs} = S_o + q_o + a_1 + aE(r)\ .$$

From (19) and (20)

Expected Profits in Period 1 E(P), will be given by,

$$E(P) = \text{Expected Sales} - \text{Expected Costs}$$

$$= E(r)\ p_o - (S_o + q_o + a_1 + a\ E(r)\ )$$

$$(21) \quad E(P) = E(r) \times (p_o - a) - (S_o + q_o + a_1)$$

In order to find the optimal values of personal selling expenditures and optimal price we assume that quality expenditures are constant. (As we saw earlier, quality changes would generally be relatively rare and of a discrete nature). The optimal values are derived treating each period separately. A separate section deals with the dynamic optimization.

Optimal Personal Selling Expenditures--1st Period Case

The optimal value of Personal Selling Expenditures (S_o*) will be a value which satisfies the equation[1]

$$\frac{dE(P)}{dS_o} = 0$$

From eq(3) and eq(21), $\frac{dE(P)}{dS_o}$ is given by,

$$(22) \quad \frac{dE(P)}{dS_o} = \sum_{n=o}^{n} \sum_{r=o}^{n} \frac{e^{-\lambda^*}(-1)(\lambda^*)^n + e^{-\lambda^*} n(\lambda^*)^{n-1}}{n!}$$

$$\times Nk \frac{S_c}{(S_o^* + S_c)^2} \times \binom{n}{r} \left(\sigma_{p_o}\right)^r \left(1-\sigma_{p_o}\right)^{n-r} r\left(p_o - a\right) - 1 = 0$$

The value of S_o* can be found from

$$(23) \quad \frac{NkS_c}{(S_o^* + S_c)^2} \sum_{n=o}^{n} \sum_{r=o}^{n} \left(\frac{e^{-\lambda^*}(-1)(\lambda^*)^n + e^{-\lambda^*} n(\lambda^*)^{n-1}}{n!} \right)$$

$$\times \binom{n}{r} \left(\sigma_{p_o}\right)^r \left(1-\sigma_{p_o}\right)^{n-r} \quad r(p_o - a) = 1$$

1. In what follows, second-order conditions will be assumed fulfilled.

Equation (23) cannot be solved by conventional methods because in the equation λ^* itself incorporates S_o^*. Hence trial and error method has to be resorted to.

In such an approach first a value of λ^* is assumed, say, $\lambda^* = \lambda_1^* = 5$. Using this value of (λ^*), equation (23) is solved for (S_o^*). This value of (S_o^*) is substituted in equation (3) and we find λ^* from

$$(24) \quad \lambda^* = NK\left(\frac{S_o^*}{S_o^* + S_c}\right)$$

If (λ^*) calculated from equation (24) is equal to 5, then the derived value of S_o^* is accepted. Otherwise, the newly calculated value (λ^*) is used in equation (23) to solve for $(S_o^*)_2$. Again this value of S_o^* is used to calculate $(\lambda^*)_2$. If this value is different from λ^*, then the process is repeated. Often only a few iterations will be necessary before convergence occurs and $(\lambda^*)_{n+1} = \lambda_n^*$. Then the value $(S_o^*)_{n+1}$, is taken as the solution to equation (23).

<u>Optimal Price--1st Period Case</u>

From equation (21) and equation (7)

$$(25) \quad E(P) = \left(\sum_{n=0}^{n} \sum_{r=0}^{n} \frac{e^{-\lambda} \cdot \lambda^{n}}{n!} \binom{n}{r} \left(\phi p_o\right)^r \left(1-\phi p_o\right)^{n-r} r(p_o-a)\right)$$

$$- \left(S_o - q_o + a_1\right)$$

$$\text{and} \qquad p_o = \left[\int_{p_o}^{\infty} \tilde{\tilde{p}}\, d\tilde{\tilde{p}}\right]^{K^1}$$

Knowing the distribution of $\tilde{\tilde{p}}$ we can evaluate $\int_{p_o}^{\infty} \tilde{\tilde{p}}\, d\tilde{\tilde{p}}$. Then the optimal value of p_o is the one which satisfies the equation

$$\frac{d(E(P))}{dp_o} = 0.$$

This equation will involve the exponent r and (n-r) as well as the summation. Therefore, the easiest method would generally be to use the equation $\frac{d \log E(P)}{d p_o} = 0$ instead of $\frac{d E(P)}{d p_o} = 0$ to solve for the optimal value of p_o.

t^{th} Period Case: Expression for λ

In the general t^{th} period, the effects of both quality and personal selling are reflected in the expression for λ. From equation (12) the general expression for λ can be written as

$$(26) \quad \lambda_t = (r_1)_t + (b_{1,t})(N-r_{t-1}) K_t (1-e^{-\frac{q_{o,t}}{q_{c,t}}})$$
$$+ (b_{2,t})(N-r_{t-1}) K_t \left(\frac{S_{o,t}}{S_{o,t}+S_{c,t}}\right)$$

Optimal Personal Selling Expenditures--t^{th} Period Case

From equation (26) and equation (2)

$$(27) \quad \frac{dE(P)_t}{dS_{o,t}} = \sum_{n=o}^{N} \sum_{r=o}^{n} e^{-\lambda_t^*}(-1)(\lambda_t^*)^n + e^{-\lambda_t^*} n (\lambda_t^*)^{n-1}$$

$$\times (b_{2,t})(N-r_{t-1}) K_t \left(\frac{S_{c,t}}{(S^*_{o,t}+S_{c,t})^2}\right) \binom{n}{r} \left(\delta p_{o,t}\right)^r \left(1-\delta p_{o,t}\right)^{n-r} \left(p_{o,t}-a_t\right) = 0$$

In this case again $(S_o^*)_t$ is found by the trial and error method described above.

Optimal Price--t^{th} Period Case

From equation (26) and (7)

$$(28)\quad E(P_t) = \left(\sum_{n=o}^{N} \sum_{r=o}^{n} \frac{e^{-\lambda_t} \lambda_t^n}{n!} \binom{n}{r} \left(\delta p_{o,t}\right)^r \left(1-\delta p_{o,t}\right)^{n-r} r(p_o - a_t) \right)$$

$$- (S_{o,t} - q_{o,t} + a_{1,t})$$

$$\text{and} \qquad p_{o,t} = \left[\int_{p_o}^{\infty} \tilde{\tilde{p}}_t \, d\tilde{\tilde{p}}_t \right] K_t^1$$

If we know the distribution of $\tilde{\tilde{p}}$, we can evaluate $\int_{p_{o,t}}^{\alpha} \tilde{\tilde{p}}_t d\tilde{\tilde{p}}_t$ and the optimal value of $p_{o,t}$ is the one which satisfies the equation

$$(29)\quad \frac{d\, E(P_t)}{d\, p_{o,t}} = 0.$$

Multiperiod Optimization

In single period optimization, the optimal values, viz S_o^* and p_o^* were derived for each period separately, without giving consideration to the effect upon consequent periods. In multiperiod optimization the firm allocates its quality expenditures, personal selling expenditures and decides on its price in such a way so that optimal payoffs are achieved not only in the very next period but over a number of periods.

In what follows we assume that the firm is planning for a horizon of three periods from the end of period t. It has available the same three instruments as before: quality expenditures, personal selling expenditures and price.

Optimal Quality Expenditures

As indicated earlier, it is unlikely that the firm could continuously vary its product quality. It would normally have at its disposal a few discrete alternatives. Let us assume that at the end of period t it has three alternatives q_{o1} (the present level), q_{o2}, and

q_{o3}. Once a decision to change is made at time t, the firm would not change its quality again for a number of periods. It will be assumed here that it will not change quality expenditures for at least three periods.[6]

The basic approach in this method would be to maximize the total payoff for all the three periods considered together rather than maximizing each one separately. The general expression for this maximization is untractable analytically. Here we show two simplified methods of dealing with the problem. In the first method we take S_o and p_o to be constant for all the three periods. In the second method S_o and p_o are changed for each period according to the single period optimization rules of equation (27) and equation (29).

<u>First Simplified Method</u>

From equation (26) the value of λ_{t+1} is given by

$$(30) \quad \lambda_{t+1} = (r_1)_t + (b_{1,t+1})(K_{t+1})(N-r_t)(1-e^{-\frac{q_{o,t+1}}{q_{c,t+1}}}) + (b_{2,t+1})(K_{t+1})\left(\frac{S_{o,t+1}}{S_{o,t+1}+S_{c,t+1}}\right)$$

In the first method $q_{o,t}$ is constant for all three periods at either one of the values q_{o1}, q_{o2}, and q_{o3}. Similarly, $S_{o,t}$ can take either one of the values S_{o1}, S_{o2}, and S_{o3} and p_c can have either one of the

[6]To simplify the treatment, we assume that keeping the present level, q_{o1}, is also a decision that holds for three periods or more. Clearly, one could analyze the situation where the <u>timing</u> of the quality change is important--such an analysis could be easily carried out as the present one for alternative t's.

values viz p_{o1}, p_{o2}, p_{o3}.[7] To maximize profits over all the three periods, the following procedure will be adopted.

1. Using given values q_{o1} and S_{o1}, the value of λ_{t+1} is determined from equation (30) (r_t in equation (30) will be the sales figure for period t).
2. Using the value of p_{o1} for price, from equation (18), (19), and (21) the expected profits $E\ (P)_{t+1,q_{o1}}$ can be calculated.
3. Updating of $b_{2,t+2}$ is done by using equation (13), and equation (30) is used to calculate λ_{t+2}. Since r_{t+1} is not known $E(r_{t+1})$ is used instead of r_{t+1}. Again, the same values of q_{o1} and S_{o1} are used for $q_{o,t+2}$ and $S_{o,t+2}$.
4. Using the value of p_{o1} for price and equation (18), (19) and (21) the expected profits, $E\ (P)_{t+2,q_{o1}}$, are calculated.
5. Part (3) and (4) are used again to calculate $E\ (P)_{t+3,q_{o1}}$.

The payoff of q_{o1}, for three periods is given by

$$(31)\quad \text{Payoff for } q_{o1} = E\ (P)_{t+1,q_{o1}} + E\ (P)_{t+2,q_{o1}} + E\ (P)_{t+3,q_{o1}}$$

Using the same procedure described above payoffs for q_{o2} and q_{o3} for all three periods are calculated (using the same values S_{o1} and p_{o1} for Personal Selling Expenditures and Price)

$$(32)\quad \text{Payoff for } q_{o2} = E\ (P)_{t+1,q_{o2}} + E\ (P)_{t+2,q_{o2}} + E\ (P)_{t+3,q_{o2}}$$

$$(33)\quad \text{Payoff for } q_{o3} = E\ (P)_{t+1,q_{o3}} + E\ (P)_{t+2,q_{o3}} + E\ (P)_{t+3,q_{o3}}$$

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[7] The assumption that instruments can take three values can of course be relaxed.

Comparing equations (31), (32), and (33), the decision can be made on whether the firm should adopt quality expenditures at q_{o1}, q_{o2}, or q_{o3} level given its Personal Selling Expenditures to be S_{o1} and price to be p_{o1} for all periods.

As mentioned earlier there are three options of personal selling expenditures that a firm can have viz S_{o1}, S_{o2}, S_{o3} and there are three price options viz p_{o1}, p_{o2}, and p_{o3}. This gives rise to 27 possible combinations of Personal Selling Expenditures, Price, and Quality. Equations (31), (32), and (33) give us the optimal value of $q_{o,t}$ to be chosen only from one set of these combinations. A firm in a dynamic situation would need to use the procedure described above to find the Payoffs over all three periods for each one of these alternatives. From amongst the alternatives it would choose the one which gives the highest payoff over all three periods.

Second Simplified Method

In this method, the values of Personal Selling Expenditures and price are not taken to be constant over all three periods but are "updated" according to the single period optimization rules of equation (27) and equation (29). The procedure here follows much the same steps as those described for the previous method.

1. The values of $p^*_{o,t+1}$ and $S^*_{o,t+1}$ are found by using equation (29) and (27) respectively. From these values and using q_{o1} for quality expenditures the value of λ_{t+1} is determined from equation (30).
2. Expected profits in period (t+1), $E'(P)_{t+1,q_{o1}}$, are calculated by using equation (18), (19), and (21).

3. The value of $b_{2,t+2}$ is derived from equation (13) equation (27) and equation (29) are used to find the values of $p^*_{o,t+2}$ and $S^*_{o,t+2}$, respectively. Again, using q_{o1}, for quality expenditures, the value of λ_{t+2} is determined from equation (30). (Since r_{t+1} is not known, $E(r_{t+1})$ is used.)
4. Expected profits in period (t+2), $E'(P)_{t+2,q_{o1}}$, are calculated using equation (18), (19), and (21).
5. Parts (3) and (4) mentioned above are again used to calculate $E'(P)_{t+3,q_{o1}}$.

The Payoff for q_{o1}, for all three periods using this method is given by,

(34) Payoff for $q_{o1} = E'(P)_{t+1,q_{o1}} + E'(P)_{t+2,q_{o1}} + E'(P)_{t+3,q_{o1}}$

The same procedure can now be used to find Payoffs for q_{o2} and q_{o3}.

(35) Payoff for $q_{o2} = E'(P)_{t+1,q_{o2}} + E'(P)_{t+2,q_{o2}} + E'(P)_{t+3,q_{o2}}$

(36) Payoff for $q_{o3} = E'(P)_{t+1,q_{o3}} + E'(P)_{t+2,q_{o3}} + E'(P)_{t+3,q_{o3}}$

Comparing equations (34), (35), and (36), the decision can be made whether the firm should adopt quality expenditures at q_{o1}, q_{o2}, or q_{o3} level.

It is clear that a similar approach could be used to generate the best alternatives $S_{o,t}$ (and $p_{o,t}$), given the derived $q_{o,t}$ and $p_{o,t}$ (and $S_{o,t}$, respectively). An iteration scheme again comparing the possible levels of quality could than be initiated, and carried through for personal selling and price, and then return for another iteration, etc. Such an approach could not guarantee that a true optimum was found, but would only select the best combination out of the given alternatives.

Many times this will clearly suffice considering the context of much decision making. On the other hand, it should be kept in mind that these iterations might not converge and could be costly. If it is decided that the single-period solutions for personal selling and price be used, making the iterations unnecessary, one would possibly save time and money, but the decisions would clearly be suboptimal from a dynamic viewpoint. If the first simplified method is used, furthermore, subsequent price and selling changes deemed necessary might easily make a quality change ineffectual.

The final balance between precision and cost will have to be arrived at for the particular base at hand. Instead of developing additional analysis at this level of generality, therefore, we will in the last section give a numerical example to show the calculations involved in more detail and to exhibit the form of some actual marketing mixes.

A Numerical Example

In this final section a numerical example of the use of the model in evaluating alternative strategies with respect to price, quality and selling effort will be given. The purpose is mainly to give a more concrete picture of the behavior of the system over time for alternative settings of the decision variables. In addition, as we will see, the numerical example will allow us to account for overall system constraints and determine the parameter values that satisfy these constraints.

The parameter values used for the example are displayed in Table 1. The number of customers was set at 30, a reasonable figure for any industrial markets, and the number of suppliers (including "our" firm) was set equal to 5. It was decided to run the model for four periods,

and to develop the tested strategies with three values (constant over time) on each of the three decision variables: One value above the industry average, one below, and one equal to the average. This made for 27 alternative strategies to compare.

A few comments on the other parameter choices are in order. It was assumed that all the firms had some selling so that the saturation level of k equaled 5. The scale factor α_6 was set rather low, however, so that k did not get close to saturation. It was uncovered in the early runs that when k does get close to saturation, the number of listings for any one competitor gets close to the total number possible (here equal to 30). This will make the Poisson assumption on the number of listings incorrect, since the distribution is effectively truncated just above the mean. This problem can be handled by replacing the Poisson by another appropriate distribution, for example, a binomial with n equal to the number of customers and p equal to λ/n, whenever k can be expected to go very high.

The positive constant M (see equation 9) was initially chosen as equal to 1.0. When the model was run using the average values on the firm's decision variables, the sales were higher than one would expect (for average marketing input levels it is reasonable to expect an approximately average market share). Accordingly, M was set at 2.184 when the second period λ (see equation 12) was computed. Since Q_0 is a proportion, so that ln Q_0 is negative, however, the equations (10) and (11) were left with M = 1.0, and here K was set equal to r, the number of buyers in the previous period. These changes made sales come out close to the average for average decision variable settings, and they also eliminated the problem of having the number of returning buyers

r_1 go negative (a less satisfactory resolution of this last problem would clearly have been to make the function (11) discontinuous at the value 0.)

The results from the runs are depicted in Tables 2 through 9 which give the sales (here equal to the number of customers buying) cumulative profit pictures at the end of each of the four periods. Overall, given that our planning horizon covers these four periods, Table 9 shows that cumulative profits are highest for the high price and quality options, but low selling effort level. This state of affairs seems to be quite common across many different industrial markets. The sales, as might be expected, are higher for a lower price, and generally for a higher selling effort. In the latter instance, however, a noteworthy feature of the model is that our high selling efforts will increase search activity and thus k, the average number of listings, but that with a high relative price we may lose customers who discover low-price competitors (see the last row of Table 8 for an example of this).

Overall it is clear that after some periods quality is the important force, followed by price, whereas selling effort quite quickly loses its force. (Note that with a one period horizon, the high selling effort is superior.) The importance of quality derives, of course, from its dual role as an inducer to trial (through word-of-mouth communication) and by making customers loyal. It should be noted, however, that the first period here does not represent the introductory period, since the b_1 and b_2 values were set at 0.6 and 0.4, respectively, in the initial period (this was done to make the illustration brief and yet show the main features of the model).

The extension of this example to evaluate other, more complex strategies is straightforward. Thus, one obvious set of alternatives would consist of different levels of a gradually diminishing selling effort over time--if feasible--and another the alternative timings of an increase in product quality from a low relative level. The hope is that the simple example presented has given the reader a fairly accurate idea about the behavior of the system over time for different levels on the decision variables.

Summary

In this paper an industrial marketing mix model incorporating price, quality, and personal selling decisions has been presented. The model was developed on the basis of established behavioral knowledge (both theoretical and empirical) about the industrial markets, and incorporates several of the major features of such markets. The optimal decision variable settings were shown to be untractable in the general case, although a highly simplified one-period optimization was possible. A numerical example displaying the main features of the model was developed and used to evaluate some alternative mix strategies.

TABLE 1

Parameter Values for the Numerical Example

Equation	
	Number of customers = $N = 30$
	Number of firms = $m = 5$
	Number of time periods in planning horizon = $T = 4$
	Number of buyers from "our" firm last period = $r = 6$ (initial value)
	Fraction attracted by quality = $b_1 = 0.6$ (initial value)
	Fraction attracted by selling effort = $b_2 = 1-b_1 = 0.4$ (initial value)
	Saturation level on k, the average number of firms listed by a customer = $k_s = 5$
(14)	Fixed costs for our firm = $a_1' = 25$ (in 1000 dollars)
(14)	Unit Variable costs for our firm = $a_2' = 35$ (in actual dollars)
(4)	The scale parameter relating k to total selling effort = $\alpha_0 = 0.001$
(4)	The slope parameter relating k to total selling effort = $\alpha_1 = 1.100$
(10)(11)	The constant in the relation determining how many will return = $K = r$ (initial value)
(9)(11)	The constant in the relation between quality expenditures and the quality index = $M = 1.0$ or 2.184 (see text)

TABLE 2

Expected Unit Sales: End of 1^{st} Period

Code: Qual = Quality expenditures, in 1000 dollars
Sell = Selling expenditures, in 1000 dollars
P = Price in actual dollars
Sales in 1000 units

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	4.30	4.01	3.68	4.79	4.44	4.05	5.22	4.81	4.36
Qual = 30	7.56	7.04	6.46	8.05	7.47	6.81	8.49	7.83	7.10
Qual = 35	10.27	9.57	8.78	10.77	9.99	9.11	11.21	10.34	9.37

TABLE 3

Expected Cumulative Profits: End of 1st Period

Profits in 1000 dollars

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	16.18	30.39	40.51	20.889	36.14	46.60	24.42	40.44	50.98
Qual = 30	76.30	101.24	119.00	81.154	106.792	124.36	84.82	110.87	128.00
Qual = 35	125.54	159.43	183.56	130.528	164.816	188.31	134.30	168.70	191.33

TABLE 4

Expected Unit Sales: End of 2nd Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=58	P=55	P=45	P=50	P=55
Qual = 25	3.74	3.42	3.07	4.29	3.91	3.50	4.74	4.31	3.84
Qual = 30	8.05	7.38	6.64	8.48	7.75	6.95	8.83	8.04	7.18
Qual = 35	11.63	10.68	9.63	11.96	10.94	9.83	12.21	11.13	9.96

TABLE 5

Expected Cumulative Profits: End of 2nd Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	21.06	45.99	62.72	31.711	59.02	76.73	39.24	68.29	86.44
Qual = 30	162.33	210.82	243.43	170.88	220.64	252.92	176.53	227.00	258.48
Qual = 35	278.15	346.54	392.73	284.741	353.49	398.26	288.60	357.19	400.15

TABLE 6

Expected Unit Sales: End of 3rd Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	3.49	3.19	2.86	3.99	3.64	3.26	4.39	4.00	3.57
Qual = 30	8.25	7.53	6.75	8.57	7.80	6.97	8.81	8.00	7.12
Qual = 35	12.19	11.16	10.03	12.36	11.28	10.10	12.47	11.35	10.13

TABLE 7

Expected Cumulative Profits: End of 3^{rd} Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P-50	P=55	P=45	P=50	P=55
Qual = 25	21.04	55.88	78.70	36.703	75.24	99.79	47.07	88.30	113.75
Qual = 30	252.37	324.17	370.99	262.31	335.76	382.10	267.81	342.08	387.28
Qual = 35	441.97	545.66	613.80	446.95	550.61	616.4[illegible]	448.14	551.08	614.08

TABLE 8

Expected Unit Sales: End of 4th Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	3.35	3.06	2.75	3.79	3.46	3.11	4.11	3.76	3.38
Qual = 30	8.36	7.62	6.82	8.58	7.81	6.97	8.74	7.93	7.06
Qual = 35	12.47	11.42	10.27	12.54	11.45	10.25	12.58	11.45	10.23

TABLE 9

Expected Cumulative Profits: End of 4^{th} Period

	Sell = 10			Sell = 15			Sell = 20		
	P=45	P=50	P=55	P=45	P=50	P=55	P=45	P=50	P=55
Qual = 25	18.16	62.62	91.50	37.50	86.91	118.28	49.44	102.42	135.18
Qual = 30	344.62	439.82	500.77	354.02	451.04	511.29	357.86	455.54	514.27
Qual = 35	611.47	751.36	841.99	612.85	751.99	839.20	609.79	747.38	830.58

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